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Development of new effective Nusselt–Reynolds correlations for air-cooling of spherical and cylindrical products

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Abstract—An effective study for determining the effective heat transfer coefficients of spherical and cylindrical bodies cooled in air-flow is provided in this paper. In this respect, both experimental and theoretical investigations were conducted. The center temperatures of spherical and cylindrical products, namely figs, tomatoes, pears, cucumbers and grapes, during air-cooling at a temperature of 4°C and at different flow velocities were measured. In the theoretical case, the effective heat transfer coefficients for spherical and cylindrical products cooled were determined by using the Dincer's models presented here. Using the obtained effective heat transfer coefficients together with some heat transfer coefficients data given in the literature, the diagrams of $Nu/Pr^{1/3}$ against Re were observed. Then, the following Nu–Re correlations were found as $(Nu/Pr^{1/3}) = 1.56Re^{0.426}$ and $(Nu/Pr^{1/3}) = 0.291Re^{0.592}$ for all the spherical and cylindrical products.

INTRODUCTION

COOLING is a significant food preservation technique which is employed to prevent the spoilage and to maintain the quality of food products [1–3].

Transient heat transfer between a solid product and a cooling medium is important in several food processing operations, such as cooling, freezing, heating, drying, etc. but mass transfer also occurs in connection with the heat transfer in some processes, e.g. drying. In recent years, significant progress has been made in understanding and analysing transient heat transfer during cooling of food products in order to design improved cooling system components and to optimize processing conditions [4–10]. However, very limited information is available in the literature on the determination of the effective thermal properties, e.g. heat transfer coefficient, thermal conductivity, thermal diffusivity, etc. of the food products subjected to cooling operations. Therefore, more attention has been given to the effective heat transfer coefficients of various products, which play an important role in cooling operations [11–13].

In the literature, many Nusselt–Reynolds correlations are proposed to estimate the heat transfer coefficients for solid objects cooled or heated in any fluid flow but these correlations lead to steady-state heat transfer. However, there is a need to develop some correlations for the effective heat transfer coefficients in the transient heat transfer case.

The principal purpose of this paper is to develop new effective Nusselt–Reynolds correlations for spherical and cylindrical products cooled with air-flow.

ANALYSIS

The formulation and modelling procedure used in this study are essentially the same as those in Dincer [5].

In order to establish the mathematical model for the support of the experimental observations, the boundary condition of the third kind in the transient heat transfer is considered for Biot numbers between 0 and 100. It is the most realistic case because it contains both the internal and the external resistances to the heat transfer from the products.

Consider the cooling of a solid spherical or cylindrical product of radius R immersed in an air-flow at constant temperature T_a, with a constant convective heat transfer coefficient h for both products. At $t = 0$, the temperature distribution is assumed to be given. This conduction problem in the spherical and cylindrical systems involves spherical symmetry and axial symmetry.

A homogeneous and isotropic solid sphere and cylinder, constant thermal properties, uniform initial temperatures, constant medium temperature, constant heat transfer coefficients, negligible internal heat generation, and heat conduction in the radial direction only are assumed.

Mathematical formulation of this heat conduction problem in the spherical and cylindrical coordinates for both products may be written in the following general form:

$$(\partial^2 T/\partial r^2)+(Z/r)(\partial T/\partial r) = (1/a)(\partial T/\partial t). \quad (1)$$

The formulation in terms of the excess temperature $\phi = T - T_a$ is:

NOMENCLATURE

a	thermal diffusivity [$m^2\ s^{-1}$]
a_w	thermal diffusivity of water at the product temperature [$0.148 \times 10^{-6}\ m^2\ s^{-1}$]
A, B, C	constants
Bi	Biot number
C	cooling coefficient [s^{-1}]
D	diameter [m]
Fo	Fourier number
h	effective heat transfer coefficient [$W\ m^{-2}\ K^{-1}$]
J	lag factor
J_0	zeroth order Bessel function of the first kind
J_1	first order Bessel function of the first kind
k	thermal conductivity [$W\ m^{-1}\ K^{-1}$]
L	length [m]
M	root of the characteristic equation for sphere
N	root of the characteristic equation for cylinder
Nu	Nusselt number
Pr	Prandtl number
r	radial coordinate
r^2	correlation coefficient
R	radius [m]
Re	Reynolds number
t	time [s]
T	temperature [°C or K]
U	flow velocity [$m\ s^{-1}$]
V	volume [m^3]
W	water content by weight, in decimal units.

Greek symbols

Γ	dimensionless radial distance
θ	dimensionless temperature
ν	kinematic viscosity [$m^2\ s^{-1}$]
ρ	density [$kg\ m^{-3}$]
ϕ	temperature difference [°C or K].

Subscripts

a	medium condition
c	cylinder
f	final
i	initial
n	refers to the nth characteristic value
s	sphere
1	refers to the first characteristic value.

$$(\partial^2\phi/\partial r^2)+(Z/r)(\partial\phi/\partial r) = (1/a)(\partial\phi/\partial t), \quad (2)$$

where $Z = 1$ for cylinder and 2 for sphere.

The boundary and inlet conditions for both the spherical and cylindrical body, in dimensionless form, are:

$$\phi(r,0) = \phi_i = T_i - T_a, \quad (3)$$

$$\phi(0,t) = \text{finite}, \quad (4)$$

$$(\partial\phi(R,t)/\partial r) = -(h\phi(R,t)/k). \quad (5)$$

The solution of the above equations can be made using general techniques, e.g. Laplace transform and separation of the variables as given in the literature [14,15]. Thus, the transient temperature distributions for the spherical and cylindrical products are in the form of the following equation:

$$\theta = \sum_{n=1}^{\infty} A_n B_n C_n, \quad (6)$$

where

$$A_n = \frac{2Bi_s \sin N_n}{(N_n - \sin N_n \cos N_n)}, \quad B_n = \exp(-N_n^2 Fo_s)$$

$$C_n = \frac{(\sin N_n\Gamma)}{(N_n\Gamma)} \quad \text{for a sphere,}$$

$$A_n = \frac{2Bi_c}{J_0(M_n)(M_n^2+Bi_c^2)}, \quad B_n = \exp(-M_n^2 Fo_c)$$

$$C_n = J_0(M_n\Gamma) \quad \text{for a cylinder.}$$

Equation (6) permits one to estimate the temperature at any point (center, half radius and surface) of both spherical and cylindrical bodies. For the center position, $C_n = 1$ due to $\Gamma = 0$. The Biot and Fourier numbers and dimensionless radial distance are defined as:

$$Bi = hR/k, \quad (7)$$

$$Fo = at/R^2, \quad (8)$$

$$\Gamma = r/R. \quad (9)$$

At the center of the infinite cylinder and of the sphere, $\Gamma = 0$. When $Fo > 0.2$, the infinite sum can be approximated by the first term of the series in equation (6) and may be represented by the following expressions:

$$\theta_c = A_{1,c}\exp(-M_1^2 Fo_c), \quad (10)$$

$$\theta_s = A_{1,s}\exp(-N_1^2 Fo_s). \quad (11)$$

Applying regression analyses using the least squares method, the dimensionless temperature distributions for both the cylindrical and spherical products are obtained in exponential form as:

$$\theta_c = J_{1,c}\exp(-C_c t), \quad (12)$$

$$\theta_s = J_{1,s}\exp(-C_s t). \quad (13)$$

The following equations are obtained by equating equations (10) and (12) and equations (11) and (13) in the consideration of $A_1 = J_1$:

$$M_1^2 Fo_c = C_c t, \quad (14)$$

$$N_1^2 Fo_s = C_s t. \quad (15)$$

The values of M_1 and N_1 are determined by means of the following characteristic equations $N_1 \cos N_1 = \sin N_1(1-Bi)$ and $M_1 J_1(M_1) - Bi J_0(M_1) = 0$. These characteristic equations can be simplified as given below [16]:

$$M_1^2 = (6Bi_c)/(2.85+Bi_c), \quad (16)$$

$$N_1^2 = (10.3Bi_s)/(3.2+Bi_s). \quad (17)$$

The experimental temperature distributions of both spherical and cylindrical products in dimensionless form are observed by using measured temperature values in the following equation and these temperature distributions are used in the regression analyses:

$$\theta = (T-T_a)/(T_i-T_a). \quad (18)$$

The following models are developed to determine the effective heat transfer coefficients for spherical and cylindrical bodies subjected to cooling, after making the required substitutions:

$$h_s = (3.2k_s R_s C_s)/(10.3a_s - C_s R_s^2), \quad (19)$$

$$h_c = (2.85k_c R_c C_c)/(6a_c - C_c R_c^2). \quad (20)$$

The thermal properties of food products strongly depend on their water contents. Therefore, the thermal conductivity and thermal diffusivity are estimated by using the following Sweat and Riedel correlations [6, 17]:

$$k = 0.148+0.493W, \quad (21)$$

$$a = 0.088\times10^{-6}+(a_w-0.088\times10^{-6})W. \quad (22)$$

EXPERIMENTAL

A forced-air cooling system was designed as the experimental apparatus, built in a Pilot Plant of the Refrigeration Technology Department and used in the experimental investigation. The complete experimental system consists of two major parts, namely a cooling chamber (test section) and a combined refrigerating unit. A schematic diagram and a photograph of the experimental apparatus are shown in Figs. 1 and 2.

Cooling operations were obtained in the test chamber (Fig. 2) with the outer dimensions of 1 × 1 × 2 m. The chamber was manufactured from 0.04 m square profiles whose surface was plated with stainless steel sheets of 0.0005 m thickness. Glass wool was filled between the inside and outside sheets to prevent heat losses. A radial fan, having a power of 1500 W and running at 2830 rpm, provided various air-flow velocities in the cooling chamber. Air was circulated through a channel of 0.28 m dia. made from PVC. A bellow was installed to absorb the fan vibrations. The temperatures of the products in the crate and that of the air at various points were measured by an Ellab CMC 821 multi-channel microprocessor device (Ellab Instruments, Copenhagen), which is capable of measuring with an accuracy of ±0.1°C. To minimize the conduction losses in these experimental investigations, the shortest temperature probes (DCK 8 copper–constantan thermocouples, 0.05 m long and 0.0012 m dia.) were used. The temperatures were read, displayed and printed every 30 s. The change of the relative humidity inside the test chamber was measured by a Squirrel Meter/Data Logger (Grant Instruments Ltd., Cambridge), having capacitive vaisala probes. The flow velocity of air over the products

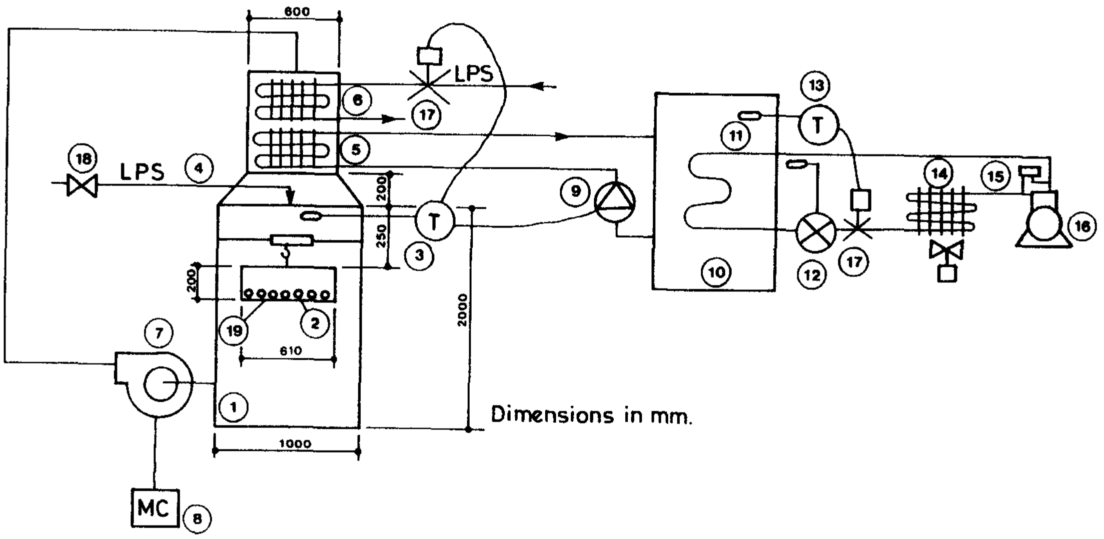

FIG. 1. Schematic diagram of experimental apparatus: (1) cooling chamber; (2) product; (3) thermostat (double); (4) low-pressure steam (LPS) inlet; (5) cold water heat exchanger; (6) steam heat exchanger; (7) radial fan; (8) fan speed controller; (9) water pump; (10) water tank; (11) evaporator; (12) thermostatic expansion valve; (13) thermostat; (14) air-cooled condenser; (15) presostat; (16) compressor; (17) solenoid valve; (18) valve; (19) crate.

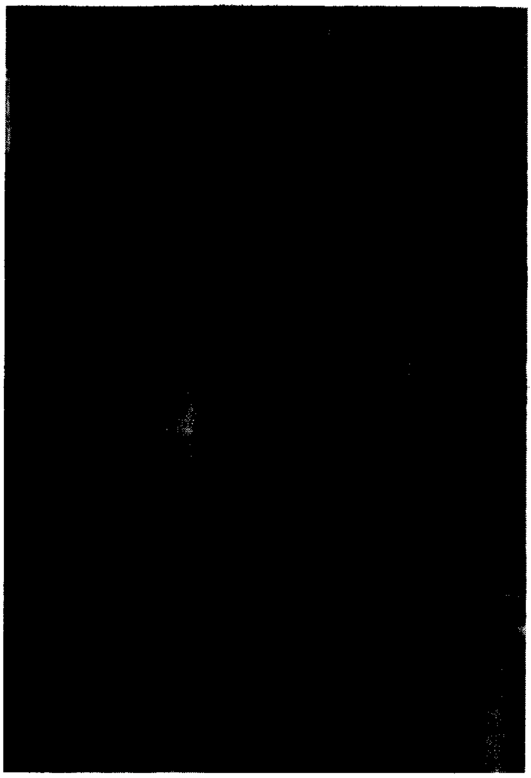

FIG. 2. A photograph showing the cabinet with the crate before commencing experiment.

inside the test chamber was measured with a digital flowmeter (Hoentzsch GmbH, Germany). The initial and final water contents of the products were measured by drying the sample in a vacuum oven at 100°C for 24 h.

The experimental studies were conducted to determine the center temperature distributions of the spherical products, e.g. tomatoes, pears, figs, and cylindrical products, e.g. cucumbers, grapes, exposed to the forced-air cooling at various air-flow velocities. For the experiments, batches of 5 kg of both spherical and cylindrical products were selected and placed into the polyethylene crates. The 12 temperature probes were embedded at the center positions of the 12 samples selected randomly in each batch. The other remaining probes were provided to measure temperatures at the bottom, in middle, and on top of the chamber, and inlet and outlet temperatures of the cooling-air. The relative humidity probes were located inside the chamber. After reaching the desired temperature and relative humidity level in the chamber, the crate containing the products of each batch was hung on the hook (Fig. 2). Then, the measurements were recorded. This procedure was repeated five times at air-flow velocities of 1, 1.25, 1.50, 1.75, and 2 m s^{-1} respectively, for each food commodity, except for figs (flow velocities of 1.10, 1.50, 1.75 and 2.50 m s^{-1} for figs). A detailed description of the experimental apparatus, instrumentation and procedure can be found in detail in Dincer and Akaryildiz [4].

RESULTS AND DISCUSSION

The center temperatures of the individual spherical and cylindrical products in the batches of 5 kg during air-cooling at various flow velocities were measured. The dimensionless temperature values were obtained using the measured temperatures of the product and the coolant in equation (18) and these dimensionless temperature distributions were regressed in the form of equations (12) and (13). Therefore, the required cooling coefficient, which is one of the most important cooling process parameters, was determined for each product. The thermal conductivities and thermal diffusivities of the products, which are heat transfer parameters, were determined using equations (21) and (22). The present models, equations (19) and (20), were used in order to determine the effective heat transfer coefficients for the individual spherical and cylindrical products. The experimental conditions included an air temperature of 4 ± 0.1°C and relative humidity of 80 ± 5%. Some heat transfer parameters and physical properties for the test samples are summarized in Table 1 and possible errors are also shown within the uncertainty bands.

The cooling coefficients of the cooling process parameters which are the function of the physical and thermal properties of the product and the effective heat transfer coefficients for the individual spherical and cylindrical products in crates containing 5 kg of

Table 1. Thermal and physical properties of the test samples

	Test samples				
	Tomatoes	Pears	Figs	Cucumbers	Grapes
L (m)	—	—	—	0.160 ± 0.0050	0.022 ± 0.001
D (m)	0.070 ± 0.0020	0.060 ± 0.0006	0.047 ± 0.001	0.038 ± 0.0010	0.011 ± 0.001
R (m)	0.035 ± 0.0010	0.030 ± 0.0003	0.0235 ± 0.0005	0.019 ± 0.0005	0.0055 ± 0.0005
ρ (kg m^{-3})	1113.62 ± 11	1229.02 ± 26	1076.0 ± 2	964.40 ± 27	1122.92 ± 24
W_i (by weight)	0.94	0.83	0.78	0.96	82.2
W_f (by weight)	0.93	0.83	0.77	0.95	82.2
T_i (°C)	21.0 ± 0.5	22.5 ± 0.5	22.2 ± 0.5	22.2 ± 0.5	21.5 ± 0.4
T_f (°C)	4.0	2.0	7.0	4.0	5.0
k (W m^{-1} °C^{-1})	0.61142	0.55719	0.53254	0.62120	0.5532
a (m^2 s^{-1})	1.444×10^{-7}	1.378×10^{-7}	1.35×10^{-7}	1.456×10^{-7}	1.353×10^{-7}

Table 2. Effective heat transfer coefficients for individual tomatoes and pears cooled with air-flow

	Tomatoes			Pears		
U (m s^{-1})	r^2	C (s^{-1})	h (W m^{-2} °C^{-1})	r^2	C (s^{-1})	h (W m^2 °C^{-1})
1.00	0.974	0.0001980	10.89±0.43	0.989	0.0002763	12.62±0.17
1.25	0.970	0.0002302	13.08±0.54	0.994	0.0003039	14.18±0.20
1.50	0.966	0.0002371	13.56±0.57	0.988	0.0003315	15.82±0.23
1.75	0.968	0.0002555	14.90±0.65	0.985	0.0003361	16.10±0.24
2.00	0.991	0.0002861	17.24±0.73	0.992	0.0003897	19.51±0.27

product during cooling in a forced-air stream at a temperature of 4°C and at different flow velocities are given in Tables 2–4.

As can be seen in Tables 2–4, the cooling coefficients vary with an increase in the flow velocity, with high correlation coefficients around 0.90. The values of the cooling coefficient were found to be highly sensitive to the size of the products and their surfaces exposed to the cooling medium. The effective heat transfer coefficients for the individual products were found to be strongly dependent on the cooling coefficients. The values of the effective heat transfer coefficients and Biot numbers, which were affected by the air-flow velocity, increased with an increase in the flow velocity from 1 to 2.5 m s^{-1} during cooling in air. The increase in the effective heat transfer coefficient was found to be 36.8% for tomatoes, 35.3% for pears, 27.3% for figs, 50.5% for cucumbers and 27.3% for grapes. The variations in the effective heat transfer coefficient and especially its increase with respect to an increase in the air-flow velocity strongly indicate that the flow and temperature profiles as well as the thermal and physical properties of the air around the product were influenced by the flow velocity and were different for each experiment. The increase in the effective heat transfer coefficient from $U = 1.75$ to 2 m s^{-1} was found to be larger than for the other flow cases, probably due to the sudden changes in the experimental cooling medium condition. As presented above, the cooling process parameters were found to be dependent upon the experimental conditions, including different flow velocities. This would seem to be due to changes in the heat transfer environment in forced-air cooling.

Table 3. Effective heat transfer coefficients for individual figs cooled with air-flow

U (m s^{-1})	r^2	C (s^{-1})	h (W m^{-2} °C^{-1})
1.10	0.990	0.0006217	23.77±0.17
1.50	0.994	0.0006677	26.16±0.20
1.75	0.995	0.0006907	27.41±0.20
2.50	0.993	0.0007828	32.71±0.27

In spite of the use of Dincer's models, there is a need to estimate the effective heat transfer coefficient for an individual product cooled in any fluid-flow in a simple and accurate form for an engineer and researcher in practice. In this respect, the diagrams of $(Nu/Pr^{1/3})$ against Reynolds number for the spherical and cylindrical products were illustrated. In these diagrams, the effective heat transfer coefficients obtained in the present study and some heat transfer coefficients, which were taken from some studies given in the literature [5–10, 18, 19], were employed. These diagrams are shown in Figs. 3 and 4. In Fig. 3, Dincer's model was used for figs, tomatoes and pears. In addition, other data were taken from Arce and Sweat [5] for apple, ASHRAE [6] for orange, Ansari [7] for apple, orange and potato, Hayakawa and Succar [8] for tomato, Ansari *et al.* [9] for apple and potato, Guemes *et al.* [10] for strawberry, Hayakawa [19] for tomato. In Fig. 4, in addition to Dincer's model used for cucumbers and grapes, data for banana and carrot were taken from Ansari and Afaq [18].

Therefore, the following correlations were obtained for spherical and cylindrical products with correlation coefficients of 0.765 and 0.993:

$$(Nu/Pr^{1/3}) = 1.560Re^{0.426}, \tag{23}$$

$$(Nu/Pr^{1/3}) = 0.291Re^{0.592}, \tag{24}$$

where $Re = (U \cdot D/\nu)$ and $Nu = (h \cdot D/k_a)$.

The use of the above correlations is very simple and easy, and these are valid for all the spherical and cylindrical products in practical applications. To give

Table 4. Effective heat transfer coefficients for individual cucumbers and grapes cooled with air-flow

	Cucumbers			Grapes		
U (m s^{-1})	r^2	C (s^{-1})	h (W m^{-2} °C^{-1})	r^2	C (s^{-1})	h (W m^{-2} °C^{-1})
1.00	0.988	0.0003957	18.22±0.76	0.998	0.0002602	30.72±0.96
1.25	0.979	0.0004251	19.86±0.82	0.999	0.0002832	33.76±0.99
1.50	0.989	0.0004504	21.31±0.97	0.998	0.0003131	37.80±1.07
1.75	0.991	0.0004800	23.06±1.06	0.999	0.0003338	40.66±1.16
2.00	0.987	0.0005367	26.56±1.09	0.998	0.0003454	42.27±1.29

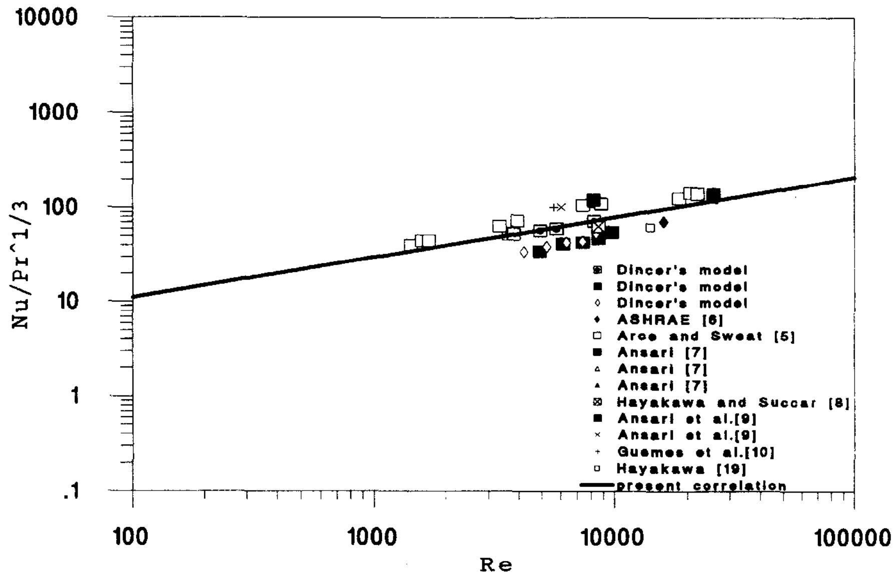

Fig. 3. The diagram of $Nu/Pr^{1/3}$ vs Reynolds number for spherical products.

an idea, it is possible to estimate the effective heat transfer coefficients without making any measurements. If we know the Reynolds number, we can estimate the effective heat transfer coefficient from the Nusselt number by using the correlations developed here. The results of this study indicated that new, simple and accurate effective Nusselt–Reynolds correlations were developed in order to estimate the effective heat transfer coefficients for all the spherical and cylindrical food products.

CONCLUSIONS

Transient heat transfer from the individual spherical and cylindrical products to the air-flow was analysed. Dincer's models were used to determine effective heat transfer coefficients for spherical and cylindrical bodies during cooling and new effective Nusselt–Reynolds correlations were developed. The heat transfer experiments were employed to measure the center temperatures of the spherical and cylindrical products, namely figs, tomatoes, pears, cucumbers and grapes during air cooling at a temperature of 4°C and at various flow velocities. These temperature data were used in the regression analyses using the least squares method in the exponential form to determine the cooling coefficients. The effective heat transfer coefficients were estimated using the values of k, a, R, and C in Dincer's models. The values of the effective heat transfer coefficients and Biot numbers,

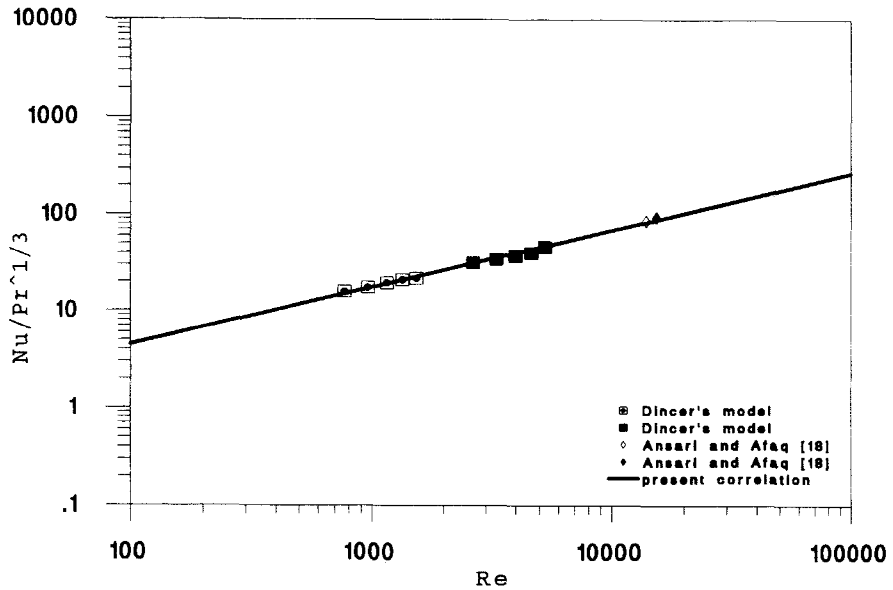

Fig. 4. The diagram of $Nu/Pr^{1/3}$ vs Reynolds number for cylindrical products.

which were affected by air-flow velocity, increased with an increase in the flow velocity from 1 to 2.5 m s^{-1} during cooling in air by 36.8% for tomatoes, by 35.3% for pears, by 27.3% for figs, by 50.5% for cucumbers and by 27.3% for grapes.

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